# SOME FACTS AND QUESTIONS AROUND THE KUZNETSOV PROBLEM

GURAM BEZHANISHVILI, DAVID GABELAIA AND MAMUKA JIBLADZE

In 1974, A. V. Kuznetsov asked whether every superintuitionistic logic is the logic of some class of topological spaces. The answer is still not known. The aim of the talk is to review some results related to it, state some of its reformulations, generalizations and particular cases that might be interesting to work on, explain some difficulties that we encountered when thinking about it, and outline some approaches to it that in our opinion might be promising.

Mostly we will focus on three themes. The first one is about possible use for the Kuznetsov problem of a Kripke incomplete superintuitionistic logic discovered in 1977 by V. Shehtman. This logic combines the Gabbay-De Jongh formula $\mathbf{bb}_2$ that on finite Kripke frames restricts possible branching to not more than two, and an intuitionistic analog of a Kripke incomplete S4-logic constructed by Fine. We are going in particular to consider the effect of these on various classes of topological spaces, including first countable, metrizable, hereditarily irresolvable, scattered and Stone spaces.

The second theme concerns versions of the Kuznetsov question for some semantics other than topological. For example, from the point of view of algebraic semantics a natural question is whether every variety of Heyting algebras is generated by complete Heyting algebras. Dually complete Heyting algebras correspond to extremally order-disconnected Esakia spaces, and in particular we will try to outline what do $\mathbf{bb}_2$ and the Shehtman axiom express for Esakia spaces.

Finally we will discuss some senses in which one might try to "stay closer" to the Kripke semantics. This includes consideration of Scott topologies on directed complete partially ordered sets, Beth and Dragalin semantics that realize complete Heyting algebras as algebras of particular upper sets (generated subframes) in an intuitionistic Kripke frame, and the analog of the Kuznetsov question for bi-Heyting algebras.

Our recent work, as well as related results by some other authors will be reviewed.