Quantified Relevant logic **RQ** with Constant Domains!? A Perspective from Quantified Modal Logics

Nicholas Ferenz

Czech Academy of Sciences, Prague, The Czech Republic
ferenz@ualberta.ca

Abstract

This work explicates and aims to solve the problems of (general frame) constant domain semantics for relevant logics, and presents some recent results concerning the Barcan formula(s) in relevant (substructural) logics with neighbourhood semantics.

The relevant and substructural logic **R** — the logic of De Morgan monoids, with an implication that rejects syntactic irrelevance — was first given a ternary relational semantics by Sylvan (né Routley) and Meyer in a series of papers [8, 9]. This semantics enjoys many of the philosophical and interpretative benefits of Kripke-style, relational frame-based semantics. To the relevantist's disappointment, the most straightforward way of generalizing this semantics to model first-order extensions of **R**, namely by adding one universal domain and interpreting the quantifiers using generalized (infinite) intersection and union, produces a semantics for which quantified **R** (**RQ**) is incomplete. The incompleteness, shown by Fine [4], was remedied (again by Fine [3]) by a genius but complicated variable domain semantics (and some additional machinery). More recently, Mares and Goldblatt [7] have developed an alternative semantics for **RQ** which employs (i) general frames, and (ii) a non-Tarskian interpretation of the quantifiers. General frames are frames built on a set of points (worlds, situations, etc) K such that it need not be that every set of points (worlds, situations, etc) can express a proposition, and so an admissible subset of $\wp(K)$, called the *admissible propositions*, is given. The Tarskian interpretation of the quantifiers uses the generalized intersection and unions, such that, given a point $a \in K$ and a variable assignment f, $a, f \vDash \forall x\mathcal{A}$ iff $a, f' \vDash \mathcal{A}$, for each f' that differs from f in at most the assignment of the variable x. The non-Tarskian interpretation of the universal quantified $\forall x\mathcal{A}$ in the Mares-Goldblatt semantics is the strongest admissible proposition that entails every instance $\mathcal{A}[\tau/x]$, where the generalized intersection of the truth sets of the instances need not be an admissible proposition.

The Mares-Goldblatt approach has been extended and employed to model a wide range of quantified modal relevant logics (Ferenz [1], Ferenz and Tedder [11, 2]), identity in relevant logics (Ferenz [1], Standefer [10]), and quantified modal classical logic (Goldblatt [5], Goldblatt and Mares [6]).

Of particular interest to the author is a handful of results in Goldblatt and Mares [6] and Goldbatt [5], which show that certain quantified modal logics are (1) incomplete with respect to the constant domain, non-general-frame semantics with a non-Tarskian interpretation of the quantifiers, (2) complete with respect to the Mares-Goldblatt semantics, but (3) complete with respect to constant domain, non-general-frame semantics with a *Tarskian* interpretation of the quantifiers. That is, we can obtain completeness for these logics without using the full power of the Mares-Goldblatt semantics.

The case for **RQ** is similar in some respects. First, the incompleteness shown by Fine shows that a constant domain, non-general semantics with Tarskian truth does not characterize **RQ**. Second, the Mares-Goldblatt semantics does in fact characterize **RQ**. What is left to show is whether or not **RQ** can be characterized by employing non-general frames with Tarskian truth conditions. The present paper aims at solving this problem.

In the classical setting, the canonical model is constructed from ω-complete theories, where *omega*-complete theories are those theories of a logic which do not contain every

instance $\mathcal{A}[\tau/x]$ of formula without also containing the universally quantified $\forall x\mathcal{A}$. The Barcan formula plays a critical role in the completeness proof, where Thomason [12] initially demonstrated that the formula ensures that certain theories obtained from collections of modal formulas are ω-complete. In this regard, the Barcan formula essentially "repairs" the completeness of some logics with respect to universal domain semantics.

For **RQ**, it is an open question whether or not any additional axioms are sufficient to repair completeness with respect to Tarskian, non-general frames. A solution to the problem aimed here — completeness for Tarskian general frames — may provide an avenue to repairing completeness in the non-general case. In particular, there may be formulas that provide a service analogous to the Barcan formula. That is, showing that a set of formulas generated from implicational formulas — by taking just the right set of antecedents or just the right set of consequents of a theory — must be ω-complete.

As both implication and modalities are treated intensionally — that is, modelled using relations between points in a frame — there are relevant questions as to the relations that hold between properties of the ternary and binary relations, the interpretation of the quantifiers, and formulas which 'mix' the various intensional operators, such as the Barcan formula. I will give independence results of several properties and Barcan formulas for neighborhood ternary relational semantics, and discuss some implications for stronger logics and philosophical perspectives.

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