Some properties of residuated lattices using two parameters derivations

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Residuation is a fundamental concept of ordered structures. Non classical logic is closely related to logical algebraic systems and it is well-known that the algebraic study of logical systems plays a significant role in artificial intelligence, formal concept analysis for example ([5],[14]). Such systems are usually modeled as partially ordered sets together with suitable operations reflecting their properties. Residuated lattices are obtained by adding a residuated monoid operation on lattices. Researches based on residuated lattices have shown them as valuable tools for solving both algebraic and logical problems ([7],[15]).

The notion of derivation, which comes from mathematical analysis, is useful for studying some structural properties of various kinds of algebra, in particular it has allowed to characterize distributive and modular lattices. Indeed it has been applied to theory of algebras with two operations + and . specially to the theory of commutative rings in 1957 [12] by Posner. For a ring $\mathcal{R} := (R; +, .)$, a map $d : R \to R$ is called a derivation if it satisfies the condition : For all $x, y \in R$,

$$\begin{aligned} d(x+y) &= d(x) + d(y) \\ d(x.y) &= d(x).y + x.d(y). \end{aligned}$$

It was applied to the theory of lattice $\mathcal{L} := (L; \vee, \wedge)$ by Szász in 1975 [13], where the operations + and . were interpreted as lattice operations $\vee$ and $\wedge$ respectively. Further, the concept of derivation is also applied to other algebra, such as BCI- algebra by Y. B. Jun and X. L. Xin in 2004 [8], later by Alsheri in MV-algebra in 2010 [2]. P. He proposed the notion of derivation in residuated lattices [6] in 2016. Based on [16], the concept of derivation was extended to f-derivation in lattices by Çeven and Özturk [4], in this work authors characterized distributive, modular lattices by using f-derivation. Maffeu et al introduced the concept of f-derivation in residuated multilattice in 2019 [11] . The concept was further explored in the form of (f, g)-derivations in lattices by Asci in 2008 [3], later by Alsatayhi on BL-algebras in 2017 [1]. In

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the same direction Keubeng et al have extended the notion of (f, g)-derivation in residuated multilattice [10].

The primary goal of this talk is to extend the notion of derivation by introducing two-parameter derivations in a bounded commutative residuated lattice. After defining this notion, we illustrate it with some examples and study the properties of some related notions. We give the condition for a (f, g)-multiplicative derivation to be monotone. Moreover, the set of fixed points is defined using the notion of (f, g)- multiplicative derivation of bounded commutative residuated lattices and the conditions for this set to be a down closed set and an ideal are given. We conclude with the characterization of set of complemented elements in terms of its derivations.

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