Truthmaker Semantics for Degreeism of Vagueness [*]

No name for blind review[1]

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Philosophers have been discussing vagueness and tackling its related paradox known as the sorites for many reasons, mostly linguistic and sometimes metaphysical and more (cf. [3]). When philosophers talk about vagueness, they often end up talking about semantics. In fact, most solutions towards the paradox revise semantics and/or logic: supervaluationism renovates semantics with supervalues, degreeism suggests many-valued logic; epistemicism is a traditionalist who keeps classical logic and semantics. In the market of semantic builders, truthmaker is a rising star with its expressive power powerful enough for hyperintensionality. Still, few have adopted truthmakers for vagueness. An exceptional case [5] suggests an argument appealing to truthmaker gaps but only for his version of epistemicism. Is there any other application of truthmaker semantics in the study of vagueness?

The goal of this paper is to offer an affirmative answer to these questions, by designing truthmaker semantics for a different position on vagueness. Among many positions at our hands, this paper works on a popular one: degreeism (degree theory). As its name tells, degreeism renews the semantic concept of truth value from binary (truth 1 and false 0 and nothing else) to many-valued (often infinite). However, importing truthmakers into degreeism is not straightforward. While truthmakers are about quality and use mereology when formalizing, degreeism is based on a quantity idea, namely a segment of real numbers [0,1]. How can we convert mereological structures of truthmaking into degreeists' real numbers?

The key idea of this transition is to import measure theory. A measure is, roughly put, a mathematical generalization of geometrical measures such as distance, length, area, and volume. This formal notion is applied to many things including physical mass and probability of events. Given degreeism is often associated with probability theory as they both feature the real fragment $[0, 1]$ as a central part of their formalization, this already seems a good match. We see an evaluation function μ that assigns a truth value to given truthmakers as a measure function, which satisfies the standard axioms of measure theory. The definition tells us how naturally these concepts fit degreeism. For one thing, an axiom says that the measure of the null set is zero,

$$\mu(\emptyset) = 0.$$

This corresponds to our intuitive idea that if a sentence has no truthmaker at all its truth value should be zero. Also, (countable) additivity confirms our idea on the relationship between truthmakers and truth values — the more truthmakers (e.g. evidence) a truth has, the more certain it is.

Having introduced truthmaker semantics for degreeism, this paper discusses the benefits of this semantics to further support how truthmakers are useful for the discussion of vagueness, at least for degreeism. This resulted semantics can resolve two formal issues of degreeism. One is about triviality [4]. Some may want to characterize vague predicates (from non-vague) by the formal concept of continuity. More technically speaking, one may want to characterize vague terms by whether its evaluation function from (a subset of) N (the number of hair) to truth values [0,1]. Unfortunately, this does not work because the domain (the number of hair, with the most natural topology) is discrete, hence any function from there is trivially continuous. In our renewed framework, such a worry disappears. Our domain is not the natural number but a

set of truthmakers, whose topology is not necessarily discrete. The other is called the problem of "penumbral connection" [1]. This problem is about how to calculate truth values of two vague clauses connected by logical connectives. What happens if two indefinite clauses (i.e. borderline cases) are connected with a conjunction, say, "This ball is purple and this ball is red"? The truth value of this sentence should be zero, i.e. definitely false because one ball cannot have different colors at the same time. But typical degreeists say it is also indefinite. Truthmakers prepare an easy way out. Just suppose that a truthmaker for being red and another truthmaker for being purple are not compatible, formally speaking, they have no overlap on each other.

The motivation originally comes from philosophical debates. Nevertheless, this work offers an insight to rather formal studies. For a broader picture, this work can bridge two different approaches towards truth — qualitative (truthmakers) and quantitative (degree theory and probability theories). Also, since truthmaker semantics has been working as a good candidate tool for relevant logic (see [2]), it may highlight the connection between degree and relevance.

References

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