Multi-type modal extensions of the Lambek calculus for structural control

Giuseppe Greco[1], Michael Moortgat[2], Mattia Panettiere[1], and Apostolos Tzimoulis[1]

[1] Vrije Universiteit Amsterdam
[2] Utrecht University

In the tradition of 'parsing as deduction', various logical calculi have been considered for applications in formal linguistics. A lively strand of research focuses on the analysis of logical systems specifically designed to model a controlled linguistic resource management [20, 16, 21, 22, 11, 1, 28]. Research on the so-called *structural control* (in combination with various modal and substructural logics) is also motivated by applications in other domains and has given rise to a rich literature in logic (see [7, 10, 12, 5, 27]).

Lambek's Syntactic Calculus [17, 18] is an early representative of substructural logic. The original Lambek calculus lacks the required expressivity to serve as a tool for realistic grammar development. The extended Lambek calculi introduced in the 1990ies enrich the type language with modalities for structural control. These modalities have found two distinct uses [16]. On the one hand, modalities can act as *licenses*, granting the applicability of so-called *structural rules* that by default would not be permitted. On the other hand, modalities can be used to *block* structural rules that otherwise would be available.

Examples of modalities as licensors relate to various aspects of grammatical resource management: multiplicity, order and structure. As for multiplicity, under the control of modalities limited forms of copying can be introduced in grammar logics that overall are resource-sensitive systems, see [26, 25, 13, 19] for some recent examples. As for order and structure, modalities may be used to license changes of word order and/or constituent structure that leave the form-meaning correspondence intact, as illustrated e.g. in [24, 3].

An example of the complementary use of modalities as blocking devices can be found in [14, 15]. The authors propose a modally-extended type language designed to simultaneously account for function-argument structure and *dependency structure*. For function-argument structure the key opposition is between a function type A/B (or $B\backslash A$) that combines with its argument B to produce an A. Dependency structures [4] on the other hand are based on the opposition between a *head* and its *dependents*; these dependents can either be *complements* selected by the head, or *adjuncts* modifying the head. In the phrase "Alice left unexpectedly", for example, the verb "left" is the head selecting for "Alice" as a complement with the *subject* role; "unexpectedly" is an adjunct modifying the head. To capture these dependency relations, [14, 15] refine the Lambek types $NP\backslash S$ and $S\backslash S$ for "left" and "unexpectedly" to $(\Diamond^{\mathrm{su}}NP)\backslash S$ and $\Box^{\mathrm{adv}}(S\backslash S)$. In general, $(\Diamond^{\mathrm{c}}A)\backslash B$ is the type for a head selecting an A complement with dependency role c, and $\Box^{\mathrm{m}}(A\backslash B)$ for an adjunct with dependency role m modifying a head A. The dependency modalities do not come with structural rules, but they have the effect of sealing off a structure consisting of a head together with its dependents as a *domain of locality*.

In this talk, we reconsider the licensing of structural rules in the light of the locality domains induced by the dependency-enhanced type language. To put the discussion in perspective, we introduce the class of multi-type logics for explicit structural control management together with their algebraic semantics, and provide proper display calculi for the basic logics and their extensions via axioms of a specific syntactic shape (the so-called *analytic-inductive axioms* [9]) in a modular fashion (e.g. preserving completeness, subformula property and cut elimination) according to the general methodology emerged in the field of structural and algebraic proof theory [2, 6, 8, 9]; in particular, all the logics considered in

[23, 14, 15] and related work, when recast as mSCLs, can profit from the pleasant proof-theoretic and model theoretic benefits that the multi-type approach brings with it.

For each $i \in I$, a *heterogeneous structural control algebra* is a structure

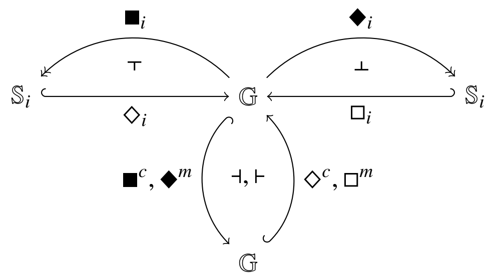
$$\mathbb{H} := (G, S_i, \Diamond_i, \blacksquare_i, \Box_i, \blacklozenge_i, \mathcal{F}, \mathcal{G}, \leq_G, \leq_{S_i})$$

such that

- $\mathbb{G} := (G, \leq_G, \mathcal{F}, \mathcal{G})$ is a partially ordered algebra, $\mathcal{F}$ (resp. $\mathcal{G}$) is a set of maps from $\mathbb{G}^n$ to $\mathbb{G}$ for some natural number n, and for each map in $\mathcal{F} \cup \mathcal{G}$ the corresponding adjoint/residual is also in $\mathcal{F} \cup \mathcal{G}$ (where the maps in $\mathcal{F}$ are left adjoints/residuals and the maps in $\mathcal{G}$ are right adjoints/residuals);

- $(S_i, \leq_{S_i})$ is a partial order; we refer to $\Diamond_i, \blacksquare_i, \Box_i, \blacklozenge_i$ as *structural control modalities*, they are unary *heterogeneous* (given their source and target do not coincide) modalities, namely such that $\blacksquare_i : G \twoheadrightarrow S_i$, $\Diamond_i : S_i \hookrightarrow G$ (where $\Diamond_i \dashv \blacksquare_i$), and $\blacklozenge_i : G \twoheadrightarrow S_i$, $\Box_i : S_i \hookrightarrow G$ (where $\blacklozenge_i \dashv \Box_i$);

- the composition $\blacksquare^c \Diamond^c : G \to G$ (resp. $\blacklozenge^m \Box^m : G \to G$) defines a closure operator, (resp. an interior operator), and the compositions $\blacksquare_i \Diamond_i : S_i \to S_i$ and $\blacklozenge_i \Box_i : S_i \to S_i$ define identity on S_i.



G is the set of *general* elements. The sort $(S_i, \leq_{S_i})$ is a set of *special* elements that witness the (controlled) licence of structural rules (that by default would not be permitted). The structural control modalities identify special elements in the general regime/type modulo the composition of adjoint pairs. For instance, in the expanded signature of the Lambek calculus the postulate $(x \otimes y) \otimes \Diamond_a \alpha \leq_G x \otimes (y \otimes \Diamond_a \alpha)$ represents a controlled form of left-to-right associativity. The x, y here are general elements, $\otimes$ is the binary fusion operator of the Lambek calculus, and $\Diamond_a \alpha$ is the image of a special element α which then *licenses* the restructuring.

In this talk the dependency modalities are *homogeneous* (as opposed to heterogeneous and given that their source and target coincide) primitive modalities defined in the general type $\mathbb{G}$ (so, they are unary maps in $\mathcal{F} \cup \mathcal{G}$). We will focus on the use of dependency modalities as means to block structural rules. Nonetheless, other design choices are also conceivable. We will also briefly expand on a few alternative design options, and we will discuss their pros and cons from the perspective of their use in linguistics.

Each and every design option falls under the scope of a general methodology that allow us to introduce multi-type proper display calculi enjoining canonical cut elimination. In particular, we observe that all the logical introduction rules are standard and reflect the minimal order-theoretic properties of the primitive operators, while the controlled linguistic resource management is explicitly captured by structural rules, so maintaining a neat division of labour that guarantees a modular treatment. At last, all the structural rules are automatically generated via the algorithm ALBA [9] (here generalized to a multi-type environment).

References

[1] G. Barry and G. Morrill, editors. *Studies in Categorial Grammar*, volume 5 of *CCS*. Edinburgh Working Papers in Cognitive Science, Edinburgh, 1990.

[2] A. Ciabattoni, N. Galatos, and K. Terui. Algebraic proof theory for substructural logics: cut-elimination and completions. *Annals of Pure and Applied Logic*, 163(3):266–290, 2012.

[3] Adriana D. Correia, Henk T. C. Stoof, and Michael Moortgat. Putting a spin on language: A quantum interpretation of unary connectives for linguistic applications. *Electronic Proceedings in Theoretical Computer Science*, 340:114–140, Sep 2021.

[4] Marie-Catherine de Marneffe, Christopher D. Manning, Joakim Nivre, and Daniel Zeman. Universal Dependencies. *Computational Linguistics*, 47(2):255–308, 2021.

[5] V. de Paiva and H. Eades. Dialectica categories for the Lambek calculus. In *International Symposium on Logical Foundations of Computer Science*, pages 256–272. Springer, 2018.

[6] N. Galatos and P. Jipsen. Residuated frames with applications to decidability. *Transactions of the American Mathematical Society*, 365(3):1219–1249, 2013.

[7] J.Y. Girard. Linear logic. *Theoretical Computer Science*, 50(1):1–101, 1987.

[8] G. Greco, P. Jipsen, F. Liang, A. Palmigiano, and A. Tzimoulis. Algebraic proof theory for LE-logics. *Submitted*, arXiv:1808.04642.

[9] G. Greco, M. Ma, A. Palmigiano, A. Tzimoulis, and Z. Zhao. Unified correspondence as a proof-theoretic tool. *Journal of Logic and Computation*, page exw022, 2016.

[10] G. Greco and A. Palmigiano. Linear logic properly displayed. *Transactions on Computational Logic: to appear*, ArXiv: 1611.04184.

[11] M. Hepple. Labelled deduction and discontinuous constituency. In M. Abrusci, C. Casadio, and M. Moortgat, editors, *Linear Logic and Lambek Calculus*, Proceedings 1993 Rome Workshop, pages 123–150. ILLC, Amsterdam, 1993.

[12] B. Jacobs. Semantics of weakening and contraction. *Annals of Pure and Applied Logic*, 69(1):73–106, 1994.

[13] Max I. Kanovich, Stepan L. Kuznetsov, and Andre Scedrov. The multiplicative-additive Lambek Calculus with subexponential and bracket modalities. *J. Log. Lang. Inf.*, 30(1):31–88, 2021.

[14] Konstantinos Kogkalidis, Michael Moortgat, and Richard Moot. Æthel: Automatically extracted typelogical derivations for Dutch. In *Proceedings of The 12th Language Resources and Evaluation Conference, LREC 2020, Marseille*, pages 5257–5266. European Language Resources Association, 2020.

[15] Konstantinos Kogkalidis, Michael Moortgat, and Richard Moot. Neural proof nets. In *CoNLL2020, Proceedings of the 24th Conference on Computational Natural Language Learning*, pages 26–40. Association for Computational Linguistics, 2020.

[16] N. Kurtonina and M. Moortgat. Structural control. In P. Blackburn and M. de Rijke, editors, *Specifying Syntactic Structures*, pages 75–113. CSLI, Stanford, 1997.

[17] Joachim Lambek. The mathematics of sentence structure. *American Mathematical Monthly*, 65:154–170, 1958.

[18] Joachim Lambek. On the calculus of syntactic types. In Roman Jacobson, editor, *Structure of Language and its Mathematical Aspects*, volume XII of *Proceedings of the Symposia in Applied Mathematics*, pages 166–178. American Mathematical Society, 1961.

[19] Lachlan McPheat, Hadi Wazni, and Mehrnoosh Sadrzadeh. Vector space semantics for Lambek Calculus with soft subexponentials. *CoRR*, abs/2111.11331, 2021.

[20] M. Moortgat. Categorial type logics. In J. van Benthem, editor, *Handbook of logic and language*, chapter 2. Elsevier, 1997.

[21] M. Moortgat and G. Morrill. Heads and phrases. Type calculus for dependency and constituent structures. *Ms OTS Utrecht*, 1991.

[22] M. Moortgat and R.T. Oehrle. Adjacency, dependency and order. In P. Dekker and M. Stokhof, editors, *Proceedings Ninth Amsterdam Colloquium*, pages 447–466. ILLC, 1994.

[23] Michael Moortgat. Multimodal linguistic inference. *Journal of Logic, Language and Information*, 5(3-4):349–385, 1996.

[24] Michael Moortgat and Gijs Wijnholds. Lexical and derivational meaning in vector-based models of relativisation. In Alexandre Cremers, Thom van Gessel, and Floris Roelofsen, editors, *Proceedings of the 21st Amsterdam Colloquium*, pages 55–64, Universiteit van Amsterdam, 2017.

[25] Glyn Morrill. Parsing/theorem-proving for logical grammar CatLog3. *J. Log. Lang. Inf.*, 28(2):183–216, 2019.

[26] Glyn Morrill and Oriol Valentín. Computational coverage of TLG: nonlinearity. *CoRR*, abs/1706.03033, 2017.

[27] Y. Venema. Meeting strength in substructural logics. *Studia Logica 54*, 54:3–32, 1995.

[28] K. Versmissen. Categorial grammar, modalities and algebraic semantics. *Proceedings EACL93*, pages 377–383, 1996.