Applications of Real Valued Logics to Probabilistic Logics

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Real–valued logics are logical formalisms whose formulas are semantically interpreted as real numbers ($\mathbb{R}$) like 0, $\frac{1}{2}$ and $\sqrt{2}$. By contrast, in Boolean logic, the truth values may only be *true* (1) or *false* (0). Examples of real–valued logics include many well–known fuzzy logics (typically interpreted in $[0,1] \subseteq \mathbb{R}$) investigated for decades in the field of mathematical logic [3], such as e.g. Łukasiewicz logic dating back to the 1930's. More examples can be found in the recent literature in computer science, where real–valued logics have been considered as formalisms for expressing and verifying properties of computer programs and interacting systems (see, e.g., [4, 5]).

In this invited talk at the LATD 2022 ("Logic, Algebra and Truth Degrees") conference, I will discuss how some old standing problems in theoretical computer science could be approached using ideas, methods and techniques developed in the field of mathematical fuzzy logic. The main problem I will discuss dates back at least to the 1982 article of Lehman and Shelah [1]:

Problem: is the SAT problem of the probabilistic logic *pCTL* decidable?

The logic pCTL ("probabilistic Computation Tree Logic" [2, §10.2]) is a logical formalism, with a Boolean semantics (*true*, *false*), for expressing properties of probabilistic transition systems (a.k.a, discrete-time Markov chains). It has important applications as a tool for specifying and verifying properties of computer programs that can use randomisation as in *probabilistic programming* [6]. It is thus remarkable that the answer to the basic problem above is still unknown.

I will argue that progress could be made by studying the problem above for a more expressive (i.e., capable of interpreting pCTL) probabilistic logic having a real–valued semantics instead of a Boolean semantics. The basic intuition is that a real–valued semantics allows for a cleaner mathematical treatment of the problems under consideration.

I will present some of the contributions I have obtained with my coauthors along this line of research. An expressive fixed–point logic, called Łukasiewicz μ-calculus [7, 9], which is capable of interpreting pCTL. A simple real–valued modal logic called *Riesz modal logic* [8], interpreted over probabilistic transition systems, which allows for an elegant, sound and complete axiomatisation. And a hypersequent calculus proof system, sound and complete for the Riesz modal logic, admitting a CUT–elimination theorem [10].

References

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