Łukasiewicz logic, MV-algebras and AF C*-algebraic truth-degrees

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As shown in [2], Łukasiewicz logic L_∞ is the only logic arising from a *continuous* $[0,1]$-valued function on the square $[0,1]^2$, having the bare minimum properties of what is usually meant by an implication on a partially ordered set of truth-degrees with a top element. Furthermore, in [4] it is shown that L_∞ -formulas code Murray-von Neumann equivalence classes of projections on those *approximately finite dimensional (AF) C*-algebras* (i.e., limits of sequences of finite-dimensional C*-algebras) whose Grothendieck group K_0 is lattice ordered. Many, if not most, preeminent AF algebras in the literature on AF-algebras have this property. For these AF-algebras, Elliott classification [1] and the Γ functor yield a one-one correspondence with countable MV-algebras, the algebras of Łukasiewicz logic. The AF algebra $\mathfrak{M}$ corresponding to the free MV-algebra F_ω on countably many generators inherits from F_ω many properties, [3]. Several uniform and non-uniform recognition problems for projections in these C*-algebras can be decided using the NP-complete logic-algorithmic machinery of Łukasiewicz logic. As shown in [4], in many relevant cases these problems turn out to be polytime decidable.

References

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