Finite Characterisations of Modal Formulas

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Motivation

We will investigate **learnability**[1] for fragments of classical modal logic, motivated by applications in description logics and database theory.

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In the literature, learnability has been studied for (unions of) conjunctive queries (ten Cate & Dalmau, 2021), XML twig-queries (Staworko & Wieczorek, 2012), LTL (Wolter, Zakharyaschev et al., 2022).

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Definition (Finite Characterisations)

A **finite characterisation** of a formula $\varphi \in \mathcal{L}$ w.r.t $\mathcal{L}$ is a pair of *finite* sets of *finite* pointed models (E^+, E^-) such that:

(i) φ *fits* (E^+, E^-) i.e. $E, e \models \varphi$ and $E', e' \not\models \varphi$ for all $(E, e) \in E^+$ and $(E', e') \in E^-$

(ii) φ is the only formula from $\mathcal{L}$ up to equivalence which fits (E^+, E^-). That is, if $\psi \in \mathcal{L}$ fits (E^+, E^-) then $\varphi \equiv \psi$.

Full Modal Logic is Fundamentally Unlearnable

Theorem

The full modal language is not finitely characterisable.

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It suffices to give one counterexample, say $\Diamond p$. Suppose that it has a finite characterisation (E^+, E^-), and let n be strictly greater than the maximum height of models in $E^+ \cup E^-$, where the height is defined as the length of the longest directed path.

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Actually, **no** formula is characterisable w.r.t. the full language.

Two Questions

A normal modal logic L is finitely characterisable if every modal formula has a fin. characterisation consisting only of models based on L-frames. Then the previous theorem says that K is not finitely characterisable.

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This can be seen as a starting observations that motivates two questions:

1. Which normal modal logics are finitely characterisable?
2. Which fragments of the modal language are finitely characterisable? (no restriction on the frame class)

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($\leftarrow$) If L is loc. tab. then it has fmp and only finitely many $\varphi_1, \ldots, \varphi_n$ up to $\equiv_L$. Then each $(\varphi_i \wedge \neg\varphi_j) \vee (\neg\varphi_i \wedge \varphi_j)$ for $i \neq j$ is sat. on a finite model $E_{i,j}$ based on an L-frame. This gives a fin. char for each φ_i.

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($\rightarrow$) If L is fin. char. then $\bot$ has a fin. char. (E^+, E^-) based on L-frames. Note that E^+ is empty so every sat. formula must be satisfied at some $(E, e) \in E^-$. If there would be infinitely many pairwise L-inequivalent formulas then cofinitely many of them $\varphi_0, \varphi_1, \ldots$ would have to be true on the same subset of E^-, and hence $(\varphi_i \wedge \neg\varphi_j) \vee (\neg\varphi_i \wedge \neg\varphi_j)$ would be unsatisfiable by properties of E^-, contradicting the fact that $\varphi_i \not\equiv_L \varphi_j$. $\square$

Modal Fragments

Definition (Modal Fragments)

For a set of connectives $C \subseteq \{\Box, \Diamond, \wedge, \vee, \top, \bot\}$, let $\mathcal{L}_C$ be the set of modal formulas built from literals in Pr using only connectives from C. Further, let $\mathcal{L}^+, \mathcal{L}^-$ denote the set of positive, resp. negative $\mathcal{L}$-formulas.

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Since the height formulas $\Box^{n+1}\bot \wedge \Diamond^n\top$ are expressible in $\mathcal{L}^+_{\Box,\Diamond,\wedge,\vee,\top,\bot}$, it follows that this fragment is not characterisable.[2] However, recent results on unions of conjunctive queries imply that $\mathcal{L}^+_{\Diamond,\wedge,\vee}$ is characterisable.

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Our main result is that $\mathcal{L}^+_{\Box,\Diamond,\wedge,\vee}$ is finitely characterisable, and this is essentially the largest characterisable (and thus learnable) fragment.

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The Positive and Monotone Modal Fragments

Kurtonina and de Rijke have semantically characterised $\mathcal{L}^{+}_{\Box,\Diamond,\wedge,\vee,\top,\bot}$ as being preserved under (directed) *simulations*, a weakening of bisimulations were the atomic clause becomes directed.

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We define a further weakening that semantically characterises $\mathcal{L}^{+}_{\Box,\Diamond,\wedge,\vee}$
Let $\circlearrowright_{\emptyset}$ denote the loop-model with empty valuation, and dually $\circlearrowright_{\mathrm{Pr}}$ is the loop-model with full valuation. Observe:

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$$\circlearrowright_{\emptyset} \not\models \text{ all } \varphi \in \mathcal{L}^{+}_{\Box,\Diamond,\wedge,\vee}, \quad \circlearrowright_{\mathrm{Pr}} \models \text{ all } \varphi \in \mathcal{L}^{+}_{\Box,\Diamond,\wedge,\vee}$$

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It follows that all $\mathcal{L}^{+}_{\Box,\Diamond,\wedge,\vee}$-formulas are satisfiable, and none are valid.

Weak Simulations

Definition (Weak Simulations)

A **weak simulation** between two pointed models $(M, s), (M', s')$ is a relation $Z \subseteq M \times M'$ such that for all $(t, t') \in Z$:

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\begin{array}{ll}
\text{(atom)} & M, t \models p \quad \text{implies} \quad M', t' \models p \\
\text{(forth}'\text{)} & \text{If } Rtu, \text{ either } M, u \underline{\leftrightarrow} \circlearrowright_{\emptyset} \text{ or } \exists u' \text{ s.t. } Rt'u' \text{ and } (u, u') \in Z \\
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Theorem (van Benthem Characterisation)

$\mathcal{L}^{+}_{\Box,\Diamond,\wedge,\vee}$ is the weak simulation-preserved fragment of FO.

Generalised Splittings

The class of pointed Kripke models with weak simulations as morphisms forms a category *wSim* (not a lattice). There is a tight correspondence between fin, char. w.r.t. $\mathcal{L}^{+}_{\Box,\Diamond,\wedge,\vee}$ and 'generalised splittings' of *wSim*.

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Definition (Generalised Splitting $\to$ not dualities!)

A **generalised splitting** of a category $\mathbb{C}$ is a pair $(\mathcal{F}, \mathcal{D})$ of finite sets of objects in $\mathbb{C}$ such that for every object E, either:

1. $\exists F \in \mathcal{F}$ such that $F \to E$ or
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where the 'or' is exclusive. In other words, the upset $\bigcup_{F \in \mathcal{F}}(F \to)$ and downset $\bigcap_{D \in \mathcal{D}}(\to D)$ partitions or 'splits' $\mathbb{C}$ in two.

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Theorem

If $\varphi \in \mathcal{L}^{+}_{\Box,\Diamond,\wedge,\vee}$ fits a gen.splitting $(\mathcal{F}, \mathcal{D})$ of wSim then $(\mathcal{F}^{+}, \mathcal{D}^{-})$ is a finite characterisation of φ w.r.t. $\mathcal{L}^{+}_{\Box,\Diamond,\wedge,\vee}$.

The main result

So the proof of our main result boils down to providing a construction for computing, for each $\varphi \in \mathcal{L}^{+}_{\Box,\Diamond,\wedge,\vee}$, a gen. splitting in *wSim* that φ fits.

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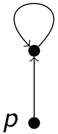

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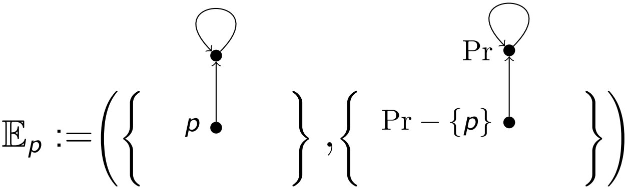
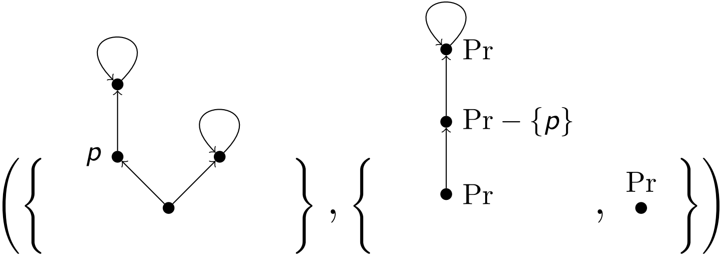


Examples

$\mathbb{E}_{\Diamond p} :=$



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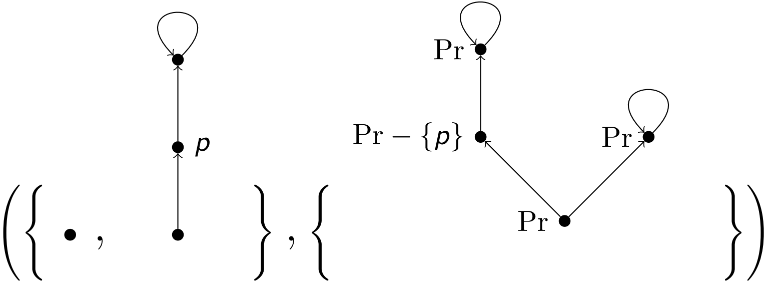
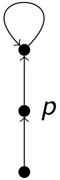
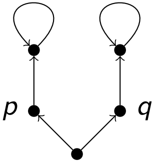
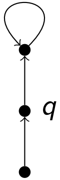


More Examples

$E^{+}_{\Box(p\vee q)} = \{\ \bullet,$ 

, 

, 

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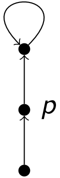
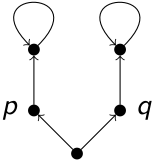
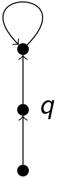
$E^{+}_{\Box(p \vee q)} = \{ \bullet ,$ 

, 

, 

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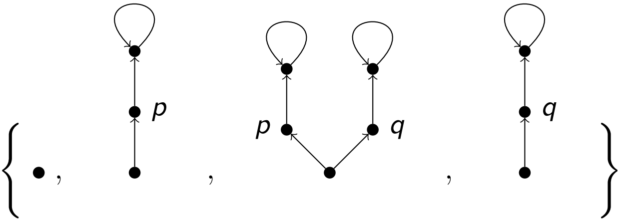

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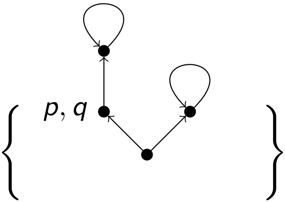

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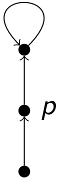
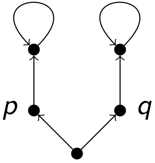
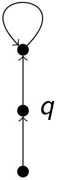
$E^{+}_{\Box(p\vee q)} = \Big\{ \bullet,$ 

, 

, 

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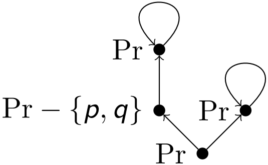
$E^{-}_{\Box(p\vee q)} = \Big\{$ 

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- *wSim* is not a lattice but does it have *multi*-products and -coproducts given some kind of bisimulations products?

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Corollaries:

- By flipping the positive and negative examples, if $\mathcal{L}$ is characterisable so is $\{\neg\varphi \mid \varphi \in \mathcal{L}\}$
- Our results extend to some uniform fragments, where variables can either occur only positively or only negatively.
- Poly-modal generalisations straightforwardly follow 'pointwise'.

Open Questions:

- Our learning algorithm is non-elementary, with a matching non-elementary lower bound. If we lose $\vee$, does the fragment $\mathcal{L}^{+}_{\Box,\Diamond,\wedge}$ become polynomially learnable?
- *wSim* is not a lattice but does it have *multi*-products and -coproducts given some kind of bisimulations products?
- Characterising modal formulas under the intuitionistic semantics?

Thanks for your attention!