Truthmakers for Degreeism in Vagueness ¹

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This talk is the first attempt of making truthmaker semantics for degreeism in the vagueness debate.

PHILOSOPHERS HAVE BEEN DISCUSSING VAGUNESS (AND ITS RE-LATED PARADOX: THE SORITES) AS A SEMANTIC AND LOGICAL DEBATE (RIGHTLY ⁴). Each philosopher has been suggesting their own logic and semantics with no interaction with their competitors (wrongly). My goal is to show truthmaker is useful for every vagueness theory as a common platform. ⁵ The current objective is to suggest a truthmaker semantics for *degreeism*, ⁶ according to which truth values are not $\{0, 1\}$ but $[0, 1] \subseteq \mathbb{R}$.

	possible worlds	truthmakers
structure	$\langle W, R \rangle$	$\langle S, \sqsubseteq \rangle$
	W: worlds, R: accessibility	<i>S</i> : states, \sqsubseteq : part-whole
expresses	intenstional	hyperintensional
based on	set theory	mereology
degreeism	probability	?

STRUCTURE. $\langle S, \sqsubseteq \rangle$ is a truthmaker frame where: *S* is a non-empty set of *states* (truthmakers) ⁷ and \sqsubseteq is a partial order on *S*, expressing its mereological (part-whole relation) structure. ⁸ From \sqsubseteq , we can define two operations: $s \sqcup t$ (fusion) ⁹ and $s \sqcap t$ (overlap) ¹⁰. *S* is closed under \sqcup , even $\emptyset \subseteq S$. $\sqcup \emptyset$ is called *null* \blacksquare . ¹¹

LANGUAGE. We consider propositions only in the form of B(n) with $n \in \mathbb{N}$. Read: "*n* pieces of sands makes a beach". ¹² The connectives are $\bigvee_{i} \wedge_{i} \neg$. ¹³

MODEL. $\langle S, \sqsubseteq, \mu_{B(n)} \rangle$ is a truthmaker model. A valuation function for each proposition $B(n) \ \mu_{B(n)} : S \mapsto [0,1]$ satisfies the followings. (TM-additivity) $\mu_{B(n)}(s \sqcup t) = \mu_{B(n)}(s) + \mu_{B(n)}(t)$ whenever there is no $s \sqcap t$. (Null) $\mu_{B(n)}(\blacksquare) = 0$. (Full) $\mu_{B(n)}(\bigsqcup S) \leq 1$.

INTERPRETATION (LOCAL). How true each truthmaker makes?

 $[s]_{Bn} = \mu_{Bn}(s),$ $[s]_{\phi \land \psi} = inf([s]_{\phi}, [s]_{\psi}),$ $[s]_{\phi \lor \psi} = sup([s]_{\phi}, [s]_{\psi}), \text{ and}$ $[s]_{\neg \phi}: \text{ undefined. } ^{14}$ Note: $[\blacksquare]_{\phi} = 0 \text{ for any } \phi.$ ¹ This handout : https://www. overleaf.com/read/sjcnvnsgwsqt ² PhD Candidate, Hitotsubashi University, Tokyo, Japan. Visiting St Andrews until September. researchmap.jp/shimpei_endo?lang=en ⊠endoshimpeiendo@gmail.com ³ The Logic Algebra and Truth Degrees (LATD) 2022/MOSAIC Kickoff Conference at Paestum, Italy

⁴ Michael Dummett. *The Logical Basis* of *Metaphysics*. Harvard University Press

⁵ See my research proposal for further details of the entire project. overleaf. com/read/hxbvpjjfjzgq ⁶ Or *degree theory*.

⁷ Intuition: think of a proper part of the world that contributes to the truth of a true statement.

⁸ See Fine and Jago's upcoming book *An Introduction to Truthmaker Semantics*. But for now:

Kit Fine. Truthmaker semantics. In Bob Hale, Crispin Wright, and Alexander Miller, editors, *A Companion to the Philosophy of Language*, pages 556 – 577. John Wiley & Sons Ltd., 2 edition

⁹ The smallest *u* such that $s \sqsubseteq u$ and $t \sqsubseteq u$

¹⁰ The biggest *u* such that $u \sqsubseteq s$ and $u \sqsubseteq t$.

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<sup>11</sup> ■ is crucial!
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¹² This is a typical case of vague terms. There is no such a thing as the smallest number of sands to make a beach. ¹³ \rightarrow is tricky.

¹⁴ I am not happy with this.

INTERPRETATION (GLOBAL). How true the entire model makes? Notation: $S_{\phi}^* = \{s^* | [s^*]_{\phi} > 0\}$. $[B(n)]] = \mu_{B(n)} \sqcup S$, $[\phi \land \psi]] = inf([\sqcup S^*]_{\phi}, [\sqcup S^*]_{\psi})$, $[\phi \lor \psi]] = sup([\sqcup S^*]_{\phi}, [\sqcup S^*]_{\psi})$, and $[\neg \phi]] = 1 - [\phi]$.¹⁵

IDEA BEHIND THESE FORMAL NOTIONS. (See the blackboard.)

..... We still need some formal works but at least this semantics reflect some degreeism's concerns.....

WHAT IS THE RESULTING LOGIC? I do not know (yet).

The Sorites solved. $[\![B(10^{24})]\!] = 1$. $[\![B(n)]\!] \neq [\![B(n-1)]\!]$ (the tolerance ¹⁶ fails!). $[\![B(0)]\!] = 0$.

PENUMBRAL CONNECTION PROBLEM? Fine ¹⁷ and others ¹⁸ does not appreciate (a version of) degreeism and blames its truth-functionality. They want: [["This ball is purple" \wedge "This ball is red"]] = 0 as the same ball cannot have two different colors at the same time. ¹⁹ To satisfy their demand, just think of $S = \{s_1, s_2\}$ and no overlap between the two. Recall that our *S* is closed under \sqcup even with \emptyset . $\sqcup \emptyset = \blacksquare$ and $[\blacksquare]_{\phi} = 0$ for any ϕ . Thus, $[\![P \land R]\!] = inf\{0, 0\} = 0$.

TRIVIAL PROBLEM? When you want to formally characterize vague terms out of non-vague ones, a natural thought is to use *continutity*. But the previous setup makes it trivial since they take a valuation function as $v : \mathbb{N} \mapsto [0,1]$ and the natural topology of \mathbb{N} is discrete — so any function from such is *trivially* continuious ²⁰. In our current truthmaker framework where the domain is a set of states *S*, it is *not* trivial. ²¹

CHARACTERIZING TRUTH-FUNCTIONALITY? Some ²² promotes a version of degreeism with *truth-functionality*. One way of accommodating this version is to think only the singleton state space $S_{SPINOZA} = \{*\}$. ²³ But is this necessary? Is there any other way of making this semantics truth-functional?

¹⁵ I am not happy with this. Standard truthmaker framework allows more flexibility with a valuation v^+ for a proposition p and another (independent) one v^- for its negation $\neg p$.

¹⁶ Roughly speaking, it says "a tiny difference does not matter". But degreeism keeps track on such a tiny difference.

18 T Williamson. Vagueness. Routledge

¹⁹ I do not find this argument convincing. But this does not matter much for my current purpose.

²⁰ Nicholas J.J. Smith. *Vagueness and Degrees of Truth.* Oxford University Press

²¹ I do not know (the natural) topology of a space of truthmakers. *Mereotopology* helps?

Roberto Casati and Achille C. Varzi. Parts and places: the structures of spatial representation. MIT Press

²² Nicholas J.J. Smith. *Vagueness and Degrees of Truth.* Oxford University Press

²³ https://www.discogs.com/release/ 22879031-John-Zorn-Spinoza