A proof-theoretic approach to ignorance

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$p :=$ Paestum is in Italy.

$Kp :=$ I know that Paestum is in Italy.

$\neg K \neg p :=$ I am ignorant that Paestum is not in Italy.

Outline

1. Introduction
2. Representing ignorance
   - Ignorance whether
   - Ignorance of unknown truths
   - Disbelieving ignorance
3. Labelled calculus *lab*WUDI
4. Conclusions

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- being ignorant of $\phi \hookrightarrow I^w\phi$
- $I^w\phi$ is $\nabla\phi$ defined by $\neg K\phi \wedge \neg K\neg\phi$.
- $\mathcal{M}, w \models I^w\phi$ iff there exists w' such that Rww' and $\mathcal{M}, w' \models \phi$ and there exists w'' such that Rww'' and $\mathcal{M}, w'' \models \neg\phi$.

System for ignorance whether

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$$\triangle\phi = \neg\nabla\phi$$

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Definition (Fan & van Ditmarsch (2015))

1. *all instances of tautologies*
2. $(\triangle(\chi \to \phi) \wedge \triangle(\neg\chi \to \phi)) \to \triangle\phi$
3. $\triangle\phi \to (\triangle(\phi \to \psi) \vee \triangle(\neg\phi \to \chi))$
4. $\triangle\phi \leftrightarrow \triangle\neg\phi$
5. *From* ϕ *and* $\phi \to \psi$ *infer* ψ
6. *From* ϕ *infer* $\triangle\phi$
7. *From* $\phi \leftrightarrow \psi$ *infer* $\triangle\phi \leftrightarrow \triangle\psi$

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- $\mathcal{M}, w \models I^u\phi$ iff $\mathcal{M}, w \models \phi$ and there exists w' such that Rww' and $\mathcal{M}, w' \models \neg\phi$

System for ignorance of unknown truths

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Definition (Steinsvold (2008))

1. *all propositional tautologies, substitution of equivalences, MP*
2. $\circ\top \leftrightarrow \top$
3. $\bullet\phi \rightarrow \phi$
4. $(\circ\phi \wedge \circ\psi) \rightarrow \circ(\phi \wedge \psi)$
5. *from* $\phi \rightarrow \psi$ *infer* $(\circ\phi \wedge \phi) \rightarrow (\circ\psi \wedge \psi)$

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- being disbelievingly ignorant of ϕ is "ϕ is true, but considered as false"
- being disbelievingly ignorant of $\phi \hookrightarrow I^d\phi$
- $\mathcal{M}, w \models I^d\phi$ iff for all $w' \neq w$ if Rww' then $\mathcal{M}, w' \models \neg\phi$ and $\mathcal{M}, w \models \phi$.

System for disbelieving ignorance

Definition

- *Axioms:*
  (Taut) All instances of propositional tautologies
  (I1) $I^d p \to p$
  (I2) $(I^d p \land I^d q) \to I^d(p \lor q)$
- *Rules: modus ponens (MP), uniform substitution (US), and*
  (IR) From $\vdash \varphi \to \psi$, *infer* $\vdash \varphi \to (I^d\psi \to I^d\varphi)$

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The operators I^d and $\Box$ are not inter-definable in standard frames, such as K, T, $S4$, $S5$ etc.

Examples

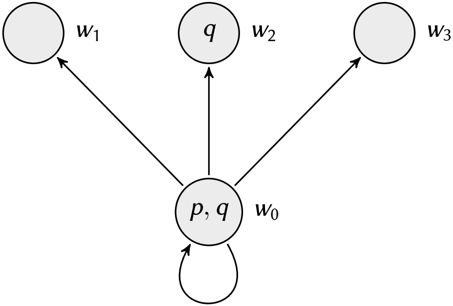

Figure: Model $\mathcal{M}_1$

$$\mathcal{M}_1, w_0 \models I^d p, \ \mathcal{M}_1, w_0 \not\models I^d q, \ \mathcal{M}_1, w_0 \not\models I^d r$$

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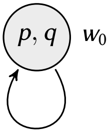

Figure: Model $\mathcal{M}_2$

$$\mathcal{M}_2, w_0 \models I^d \top$$

Two-worlds property

An accessibility relation R satisfies the two-worlds property iff for all $w \in W$, there is a $w' \in W$ such that wRw' and $w \neq w'$.

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Three operators for ignorance

- I^w, I^u, and I^d represent different aspects of the polysemic notion of ignorance. From this perspective, these three types of ignorance should coexist in the same formal setting.

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We provide a labelled sequent calculus, and prove its soundness and completeness.

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Our proposal

$$\phi \ ::= \ p \mid \bot \mid \phi \rightarrow \phi \mid \Box\phi \mid I^w\phi \mid I^u\phi \mid I^d\phi$$

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$$\phi ::= p \mid \bot \mid \phi \to \phi \mid \Box\phi \mid I^w\phi \mid I^u\phi \mid I^d\phi$$

Ignorance models

$\mathcal{M} = \langle W, R, v \rangle$:

- $W \neq \emptyset$ set of possible worlds
- $R \subseteq W \times W$
- $v : Atm \to \mathcal{P}(W)$

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R satisfies the two-worlds property:
for all $x \in W$, there is a $y \in W$ such that xRy and $x \neq y$.

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$$\Box_L \frac{xRy, x : \Box\phi, y : \phi, \Gamma \Rightarrow \Delta}{xRy, x : \Box\phi, \Gamma \Rightarrow \Delta} \qquad \Box_R \frac{xRy, \Gamma \Rightarrow \Delta, y : \phi}{\Gamma \Rightarrow \Delta, x : \Box\phi} *$$

$*$: y, z are fresh, i.e., they do not occur in $\Gamma \cup \Delta$.

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Derivation example

$$
\rightarrow_R\,\dfrac{
\wedge_L\,\dfrac{
I^u_R\,\dfrac{
I^d_{R1}\,\dfrac{
\text{init}\,\dfrac{}{x : I^w p, x : I^d p, x : p \Rightarrow x : p}
}{x : I^w p, x : I^d p \Rightarrow x : p}
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I^w_L\,\dfrac{
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}{x : I^w p, x : I^d p \Rightarrow x : I^u p}
}{x : I^w p \wedge I^d p \Rightarrow x : I^u p}
}{\Rightarrow x : I^w p \wedge I^d p \rightarrow I^u p}
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Do not apply 2w to $\Gamma \Rightarrow \Delta$ if for any x in $\Gamma \Rightarrow \Delta$ either:

(a) xRy and $x \neq y$ are in Γ for some y; or
(b) zRx and $z \neq x$ are in Γ, for some z such that $For(z) = For(x)$.

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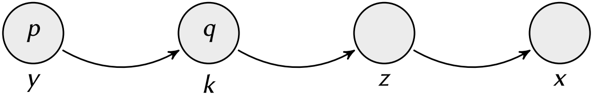

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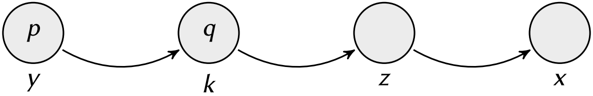

Theorem (Termination)

Root-first proof search for a sequent $\Rightarrow x : \phi$ comes to an end in a finite number of steps.

Main results, II

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If ϕ is valid, there is a derivation of $\Rightarrow x : \phi$.

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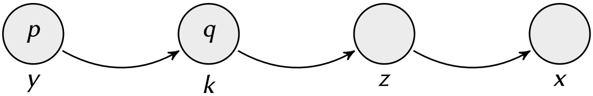

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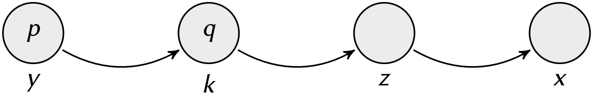

- Whenever x does not meet the two-worlds condition, and for some z we have zRx and $z \neq x$ in Γ and $For(z) = For(x)$, add $(x, z) \in \mathcal{R}^{\mathcal{B}}$.

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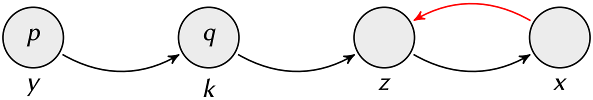

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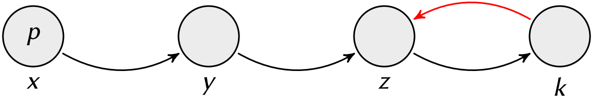
Example

$$
{\scriptstyle \to_{\mathrm{R}}}\dfrac{{\scriptstyle I^d_{\mathrm{L1}}}\dfrac{{\scriptstyle 2\mathrm{w}}\dfrac{{\scriptstyle \to_{\mathrm{L2}}}\dfrac{{\scriptstyle 2\mathrm{w}}\dfrac{{\scriptstyle 2\mathrm{w}}\dfrac{{\scriptstyle \text{fail}}\dfrac{}{xRy, x \neq y, yRz, y \neq z, zRk, z \neq k, x : p, x : I^d p \Rightarrow x : \bot, y : p}}{xRy, x \neq y, yRz, y \neq z, x : p, x : I^d p \Rightarrow x : \bot, y : p}}{xRy, x \neq y, x : p, x : I^d p \Rightarrow x : \bot, y : p}}{xRy, x \neq y, x : p, x : I^d p \Rightarrow x : \bot}}{x : p, x : I^d p \Rightarrow x : \bot}}{x : I^d p \Rightarrow x : \bot}}{\Rightarrow x : I^d p \to \bot}
$$



$$W = \{x, y, z, k\};\ R = \{(x, y), (y, z), (z, k), (k, z)\};\ v(p) = \{x\}.$$

Outline

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Questions?