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CREATIVITY AND EFFECTIVE INSEPARABILITY IN DOMINICAL CATEGORIES

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We recall the definition of a dominical category. Let C be a pointed category with a functor x: CxC $\to$ C satisfying the following

(N_1) $\phi \times \psi = 0$ iff $\phi = 0$ or $\psi = 0$;

(N_2) x restricts to $C_T x C_T \to C_T$, where it is a product, equipped with projections $X_1 \xleftarrow{p_1} X_1 \times X_2 \xrightarrow{p_2} X_2$, and the diagonal $\Delta_X: X \to X \times X$, i.e., the unique morphism such that $p_1\Delta_X = 1_X$, $p_2\Delta_X = 1_X$;

(N_3) the associativity and simmetry morphisms of this restriction are natural on CxCxC and CxC, so that x is coherently associative and symmetric;

(N_4) for each $\phi: X \to X'$ and each Y $p_1(\phi \times Y) = \phi p_1$ and $(\phi \times \phi)\Delta_X = \Delta_X\phi$.

Here C_T is the subcategory of C having the same objects and as morphisms the total morphisms of C, that is, the morphisms ϕ such that for each α, $\phi\alpha = 0$ implies $\alpha = 0$. A bifunctor x that satisfies (N_1) - (N_4) is called a near-product in C. A dominical category is a pointed category equipped with a near-product.

A <u>semigroupoid</u> is a category in which each pair of objects is isomorphic. A Turing morphism in a dominical semigroupoid C is a morphism $\tau: X \times Y \to Z$ such that for each $\phi: X \times Y \to Z$ there is a total $g: X \to X$ such that $\phi = \tau(g \times X)$. Since all pairs of objects in C are isomorphic we may restrict our attention to the case $\tau: X \times X \to X$.

A <u>recursion category</u> $\mathbb{C}$ is a dominical semigroupoid equipped with a Turing morphism. Hereafter, the notation "$\mathbb{C}$" always denotes some recursion category. An <u>index in $\mathbb{C}$</u> of $\phi: X \to X$ <u>relative to</u> τ

is a total $g: X \to X$ such that $\phi p_2 = \tau(g \times X)$.

To represent adequately the generalized incompleteness theorem of Gödel in purely algebraic terms, the notion of creative set and effectively inseparable sets ought to have algebraic (i.e. category-theoretic) representatives in $\mathbb{C}$. Accordingly, we first recall that the domain ε in X of $\phi: X \to Y$ is the compsition $p_2\langle\phi, X\rangle = p_1\langle X, \phi\rangle : X \to X$, where in general $\langle\phi, \psi\rangle = (\phi \times \psi)\Delta_X$. A domain δ in X ($\delta \in \mathrm{Dom}\, X$) is creative if there is a total $k: X \to X$ such that for all $\varepsilon \in \mathrm{Dom}\, X$ and all indices g of ε, if $\delta\varepsilon = 0$, then $\delta k g = 0$ and $\varepsilon k g = 0$.

THEOREM 1. $\mathrm{dom}(\tau\Delta)$ is creative.

In a pair (δ, ε) of domains in $X \times X$ is effectively inseparable if there is a total $k: X \times X \to X \times X$ such that for all $\delta', \varepsilon' \in \mathrm{Dom}\, X \times X$ and all indices g, h of ε', δ' respectively, if $\delta \leq \delta'$, $\varepsilon \subseteq \varepsilon'$ and $\delta'\varepsilon' = 0$, then $\varepsilon' k\langle g, h\rangle = 0 = \delta' k\langle g, h\rangle$.

A dominical category C is +-dominical if C has a coproduct, if (i) f+g is total when f and g are total, and (ii) all the morphisms $(X \times Y_1) + (X \times Y_2) \xrightarrow{(X \times i_1,\; X \times i_2)} X \times (Y_1 + Y_2)$ are isomorphisms. A section of $\phi: X \to Y$ is a $\sigma: Y \to X$ such that $\phi\sigma = \mathrm{dom}\,\sigma$ and $\phi\sigma\phi = \phi$.

The axiom of choice in a dominical category C is the assertion that every morphism has a section. If C satisfies the axiom of choice it is c-dominical; C is c+-dominical if it is both c-dominical and +-dominical.

THEOREM 2. If C is c+-dominical, then there are effectively inseparable pairs (δ, ε) in $\mathrm{Dom}\, X \times X$.