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ON THE ABSTRACT MODEL-THEORETIC NEIGHBOURHOOD OF THE LOGICS OF COMPUTERS LANGUAGES II. BASIC PROPERTIES

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Aims. The logics of computers languages (i.e. the logics of programs and the programming logics) generally does not fall into the usual abstract model-theoretic definition of "logic" (see [**BF** *I.3* and *II.1*]). The aims of this study are *(i)* the analysis of this fact and *(ii)* the evaluation of its impact on the relationships between the properties of logics.

Examples. First we list some crucial examples of logics of computers languages.

$\mathscr{L}^{fin}_{\omega\omega}$:= the first-order logic restricted to the finite structures (see [**Gu**]).

$\mathcal{PL}$:= the logic $\mathcal{H}orn_{\omega\omega}$ restricted to the countable reachable structures.

$\mathscr{L}^{re}_{\omega\omega}$:= the first-order logic restricted to the countable reachable structures.

$\mathscr{L}_e$:= the effective logic (i.e. the logic defined on the countable reachable structures by a forcing relation which forces the r. e. basic sentences only; see [**Vi**]).

The usual basic properties and the logics of computers languages. Referring to [**BF**, *II.1,XIX.2*] for the definition of the basic properties of model-theoretic logics, we have

	$\mathscr{L}^{fin}_{\omega\omega}$	$p\ell$	$\mathscr{L}^{rc}_{\omega\omega}$	$\mathscr{L}_e$
isomorphism	yes	yes	yes	yes
renaming	yes	yes	yes	yes
reduct	yes	no	no	no
atom	yes	yes	yes	no
$\neg$- *closure*	yes	no	yes	yes
$\wedge$ - *closure*	yes	yes	yes	yes
$\exists$ - *closure*	yes	yes	yes	yes
substitution	yes	no	no	no
relativization	yes	yes	yes	no

Thus in the abstract model-theoretic neighbourhood of the logics of computers languages, the notion of "logic" cannot be defined by using the reduct, atom, and $\neg$-closure properties. (Compare with [**BF**, *XIX.2* and *XIV.5*].)

Other basic properties. Let $\mathscr{L}$ be a logic (possibly a logic of computers languages) and let L,H,K be $\mathscr{L}$-vocabularies. Then H *occurs* in φ if, for every K, $\varphi\in\mathscr{L}[K]$ implies $H\subseteq K$. φ *depends (only)* on H if, for every $K\supseteq H$ and every $\mathfrak{A},\mathfrak{B}\in str_{\mathscr{L}}(K)$, $\mathfrak{A}\restriction H \cong \mathfrak{B}\restriction H$ implies $\mathfrak{A}\restriction H\models_{\mathscr{L}}\varphi$ iff $\mathfrak{B}\restriction H\models_{\mathscr{L}}\varphi$.

$occ(\mathscr{L})$:= the smallest cardinal κ such that, for every $\mathscr{L}$-sentence φ, the vocabulary H which occurs in φ has cardinality $<\kappa$ (or ∞ if no such κ exists).

$dep(\mathscr{L})$:= the smallest cardinal κ such that, for every $\mathscr{L}$-vocabulary H and every $\varphi\in\mathscr{L}[H]$, there is an $\mathscr{L}$-vocabulary $L\subseteq H$ of cardinality $<\kappa$ and a sentence $\psi\in\mathscr{L}[L]$ which is $\mathscr{L}$-equivalent to φ and depends only on L (or ∞ if no such κ exists).

$\mathscr{L}$ is *occurrence normal* if, for every $\varphi\in\mathscr{L}[L]$ which depends only on $H\subseteq L$, there is a $\psi\in\mathscr{L}[H]$ $\mathscr{L}$-equivalent to φ. $\mathscr{L}$ has *understoods* if there is an $\mathscr{L}$-sentence which has an occurrence vocabulary strictly contained in its dependence vocabulary. $\mathscr{L}$ is *regular* if has all the usual basic properties. $\mathscr{L}$ is [*weakly*] *monotonic* if, for every set Φ,Ψ of $\mathscr{L}$-sentences [with the same occurrence vocabulary], $\Phi\subseteq\Psi$ implies $Th_{\mathscr{L}}(\Phi)\subseteq Th_{\mathscr{L}}(\Psi)$.

Results. The position of the above properties in the abstract model-theoretic neighbourhood of the logics of computers languages is partially specified by the following results.

Result 1. *Let $\mathscr{L}$ be a logic with substitution such that $dep(\mathscr{L})\leqslant occ(\mathscr{L})$. Then there is a logic $\mathscr{L}^*$ with $dep(\mathscr{L}^*) = occ(\mathscr{L}^*)$ which is equivalent to $\mathscr{L}$.*

Result 2. *Let $\mathscr{L}$ be a logic with reduct. The $\mathscr{L}$ is a logic without understoods such that $dep(\mathscr{L})\leqslant occ(\mathscr{L})$.*

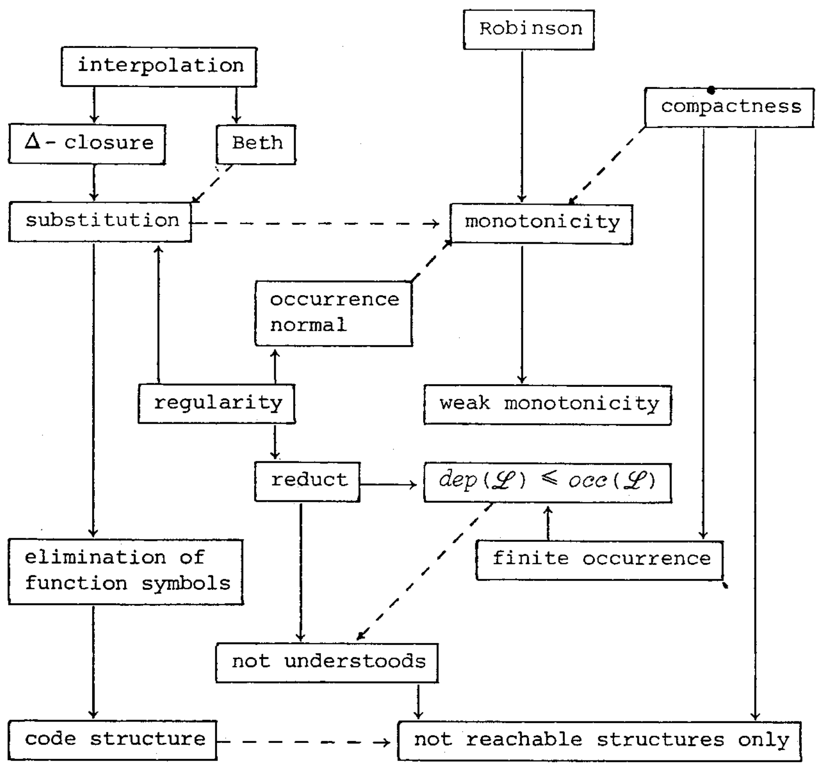
Result 3. *Every Δ- closed logic has a non-reachable structure.*

Result 4. *Every compact logic has a non-reachable structure.*

Result 5. *$\mathscr{L}_e$ and $\mathscr{L}^{rc}_{\omega\omega}$ are two weakly monotonic but not monotonic logics.*

Result 6. *Every logic with the Robinson consistency property is monotonic.*

In particular, we have that the logic $\mathscr{L}^{rc}_{\omega\omega}$ has nor the compactness neither the Δ- closure properties. Thus the restriction to the reachable structures generates the same problems of the restriction to the finite ones (compare with [**Gu**]). The relationships between compactness and monotonicity on one hand, and between regularity and monotonicity on the other, are still unknown. Hence keeping in mind the results contained in [**BF**, *XVIII*, *XIX*], we have the situation pictured in the following schema



(where - - - → means "open").

References

[BF] Barwise J., Feferman S. (eds.): *Model-Theoretic Logics*. Springer 1985.

[Gu] Gurevich Y.: *Toward logic tailored for computational complexity*. In *Proceeding Logic Colloquium '83, Aachen*. SLNM 1984.

[Vi] Vincenzi A.: *Effective logic*. In preparation.