Lezione del 23/11/23

Calcolare $\underline{1^2 + 2^2 + 3^2 + \dots + n^2} = \sum_{i=1}^{n} i^2 = \frac{1}{6} n (n+1)(2n+1)$

Ricordate che $\boxed{\sum_{i=1}^{n} i = 1+2+3+\dots+n = \frac{n(n+1)}{2}}$

$(a+1)^3 = a^3 + 3a^2 + 3a + 1$

$$2^3 = (1+1)^3 = 1^3 + 3 \cdot 1^2 + 3 \cdot 1 + 1$$
$$3^3 = (2+1)^3 = 2^3 + 3 \cdot 2^2 + 3 \cdot 2 + 1$$
$$4^3 = (3+1)^3 = 3^3 + 3 \cdot 3^2 + 3 \cdot 3 + 1$$
$$\vdots$$
$$(n+1)^3 = n^3 + 3n^2 + 3n + 1$$

$$2^3 + 3^3 + 4^3 + \dots + (n+1)^3 = (1^3 + 2^3 + \dots + n^3) + 3(1^2 + 2^2 + \dots + n^2) + 3(1+2+3+\dots+n) + n$$

$$\cancel{2}^3 + \cancel{3}^3 + 4^3 + \dots + \cancel{n^3} + (n+1)^3 = \left(1^3 + \cancel{2^3} + \cancel{\dots} + \cancel{n^3}\right) + 3\left(1^2 + 2^2 + \dots + n^2\right) + 3\left(1 + 2 + 3 + \dots + n\right) + n$$

$$\sum_{i=1}^{n} (i+1)^3 = \sum_{i=1}^{n} i^3 + 3\sum_{i=1}^{n} i^2 + 3\sum_{i=1}^{n} i + n$$

$$(n+1)^3 = 1 + 3\left(\underline{1^2+2^2+\dots+n^2}\right) + 3\left(\frac{n(n+1)}{2}\right) + n$$

$$3\left(1^2+2^2+\dots+n^2\right) = (n+1)^3 - 1 - 3\left(\frac{n(n+1)}{2}\right) - n$$

$$\underline{1^2+2^2+\dots+n^2} = \frac{1}{3}\left[(n+1)^3 - 1 - 3\left(\frac{n(n+1)}{2}\right) - n\right]$$

$$= \frac{1}{3}\left[(n+1)^3 - \frac{3n}{2}(n+1) - (n+1)\right]$$

$$= \underline{\frac{1}{3}(n+1)\left((n+1)^2 - \frac{3n}{2} - 1\right)}$$

$$= \frac{1}{3}(n+1)\left(n^2 + 2n + \cancel{1} - \frac{3}{2}n - \cancel{1}\right) = \underline{\frac{1}{3}(n+1)\left(n^2 + \frac{1}{2}n\right)}$$

Calcolare $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \dots + n^3$