

$$\mathcal{L} = \{p, q, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,)\}$$

$$\wedge, \vee, \neg, \rightarrow, \leftrightarrow$$

- $\wedge C p$

-

$$v: \mathcal{L} \longrightarrow \{0, 1\}$$

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

a) $\neg(A \rightarrow A)$ **C**

b) $A \rightarrow (B \vee A)$ **T**

c) $(A \vee B) \rightarrow (\neg A \wedge C)$

2)

A	$A \rightarrow A$	$\neg(A \rightarrow A)$
0	1	0
1	1	0

b)

A	B	$B \vee A$	$A \rightarrow (B \vee A)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

c)

A	B	C	φ $A \vee B$	\neg $\neg A \wedge C$	$p \rightarrow \varphi$
0	0	0	0	0	1
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	0	0
1	1	0	1	0	0
1	1	1	1	0	0

2) $\Gamma \quad \exists v \text{ t.c. } v(\varphi) = 1$
 $\forall \varphi \in \Gamma$

a) $\{A \rightarrow B, (A \vee B) \wedge \neg(A \vee B)\}$ NO

b) $\{C \rightarrow B, A \vee \neg B, \neg(C \wedge A), C\}$

c) $\{\underline{A} \rightarrow B, \underline{B} \rightarrow C, \underline{C} \rightarrow D, \underline{D} \rightarrow \neg A\}$

d) $\{\underline{A_0} \rightarrow B_0, \underline{A_0} \wedge A_1 \rightarrow B_0 \vee B_1, \dots$
 $\dots, \underline{A_0} \wedge \dots \wedge A_n \rightarrow B_0 \vee \dots \vee B_n, \dots\}$

3) Γ INS. DI FORMULE
 φ FORMULA

$\Gamma \models \varphi :=$ OGNI v CHE SODDISFA Γ ,

SODDISFA ANCHE φ

$$a) p \vee q \models p \wedge q$$

$$b) p \wedge q \models p \vee q$$

$$4) \varphi \equiv \neg \varphi \quad := \quad v(\varphi) = 1 \text{ sss } v(\neg \varphi) = 1$$

$$\Leftrightarrow \varphi \models \neg \varphi \quad \neg \varphi \models \varphi$$

$$a) A \rightarrow B, B \rightarrow A$$

$$b) \underbrace{(A \rightarrow B) \rightarrow A}_{\varphi}, A$$

$$\text{ss } v(A) = 1, \text{ allora } v(\varphi) = 1$$

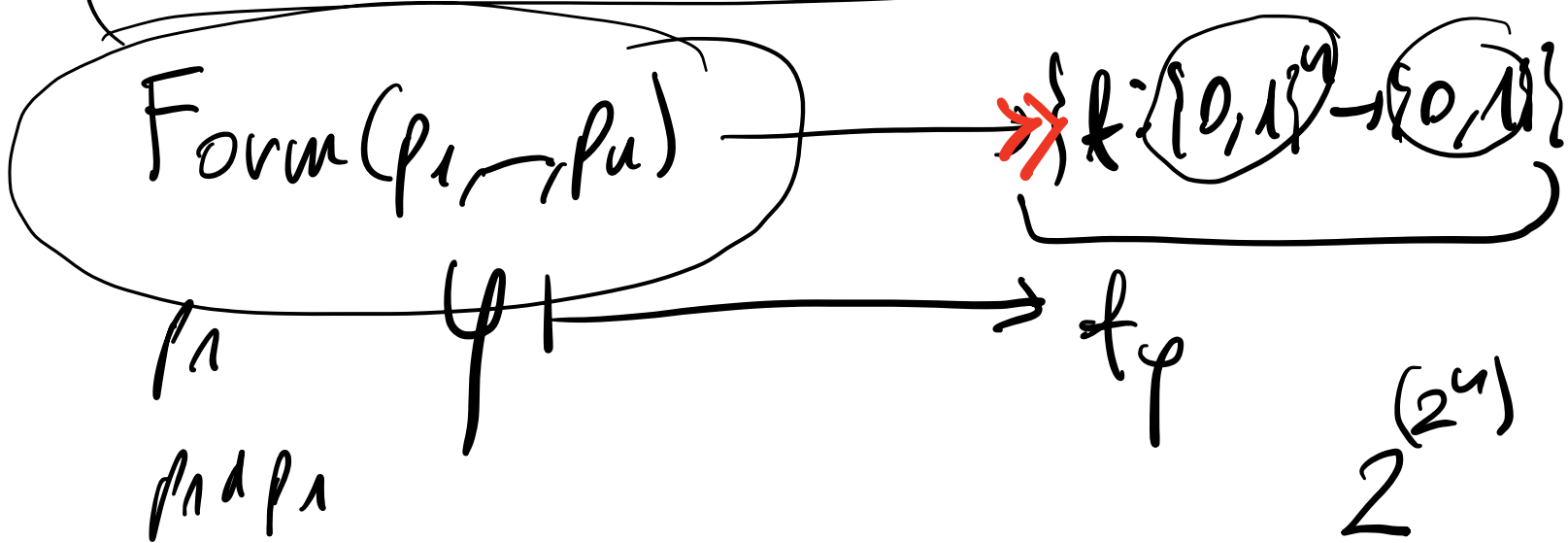
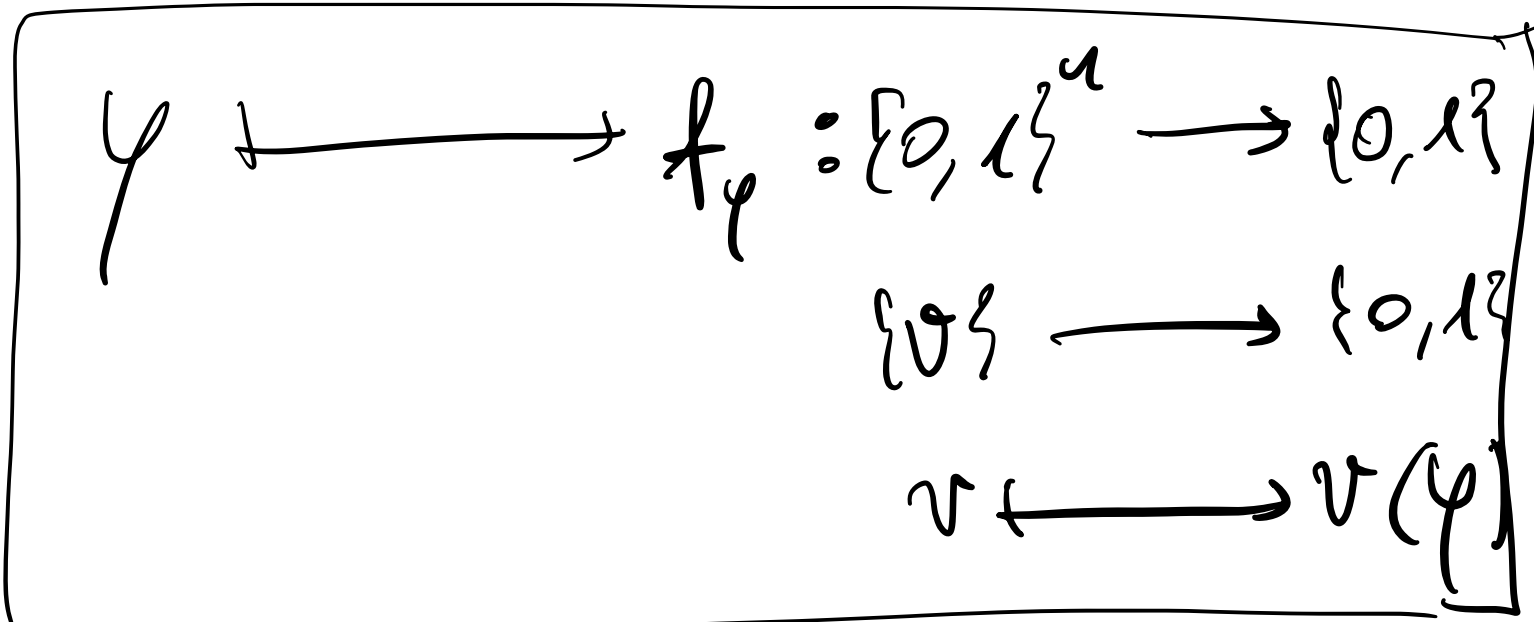
$$\text{ss } v(A) = 0, \text{ allora } v(\varphi) = 0$$

$$v(\varphi) = 1$$

$$v(\varphi) = 0$$

$\varphi(p_1 \rightarrow p_2)$

$v: \{p_1, p_2\} \rightarrow \{0, 1\}$



$p_1, p_2, \dots, p_n \quad \text{---} \quad A, B$

$\text{Form}(p_1 \rightarrow p_2)$ \cong $\{ \text{FUNZ. DI VERITA' } \}$

$\varphi \in \mathbb{R}$ IN FNC SE

$$\varphi = \varphi_1 \Lambda - \Lambda \varphi_k$$

$$\varphi_i = \alpha_{i1} V \rightarrow V \alpha_{m2}$$

$$\alpha_{ij} = x_{ij} \text{ OPPURE } \neg x_{ij},$$

x_{ij} VARIABILI

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$$\varphi = \varphi_1 V - V \varphi_k$$

$$\varphi_i = \alpha_{i1} \Lambda \rightarrow \Lambda \alpha_{m2}$$

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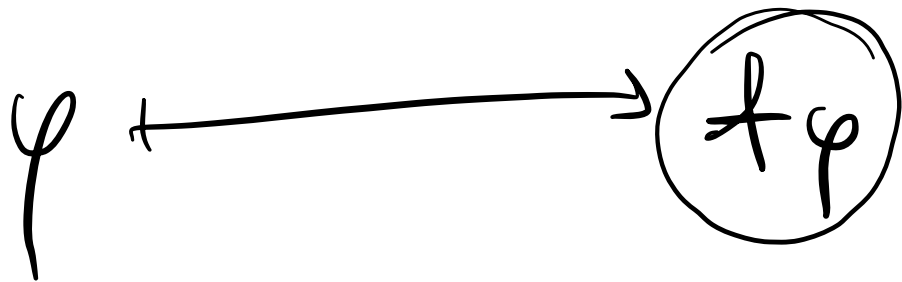
x_{ij} VARIABILI

Forma $(p_1 \rightarrow p_n) \rightarrow \{k: \{0,1\}^n \rightarrow \{0,1\}\}$

φ
+ - - - $\rightarrow f$

FND

FNC



$\varphi(p_1 \rightarrow p_n)$

FND: $\forall v \gamma_{v,i} := \begin{cases} p_i & \text{se } v(p_i) = 1 \\ \neg p_i & \text{se } v(p_i) = 0 \end{cases}$

$$\chi_v := \bigwedge_{i=1}^n \chi_{v,i}$$

$$\chi := \bigvee_{v \text{ T.C.}} \chi_v$$

$$\sigma(\chi) = 1$$

$$a) \quad \neg a \leftrightarrow b = \varphi(a, b)$$

	a	b	$\neg a$	$\neg a \leftrightarrow b$
v_1	0	0	1	0
v_2	0	1	1	1
v_3	1	0	0	1
v_4	1	1	0	0

$$\chi_{v_2} = \neg a \wedge b$$

$$\chi_{v_3} = a \wedge \neg b$$

FND

$$\varphi = \underline{(\neg a \wedge b)} \vee \underline{(a \wedge \neg b)}$$

$$\varphi(p_1 \rightarrow p_n)$$

FNC: $\forall v \tilde{\varphi}_{v,i} := \begin{cases} p_i & \text{ss } v(p_i) = 0 \\ \neg p_i & \text{ss } v(p_i) = 1 \end{cases}$

$$\tilde{\varphi}_v := \bigvee_{i=1}^n \tilde{\varphi}_{v,i}$$

$$\tilde{\varphi} := \bigwedge_{v \text{ T.C.}} \tilde{\varphi}_v$$

$v(p) = 0$

$$a) \neg a \leftrightarrow b = \varphi(a, b)$$

	a	b	$\neg a$	$\neg a \leftrightarrow b$
v_1	0	0	1	0
v_2	0	1	1	1
v_3	1	0	0	1
v_4	1	1	0	0

$$\overline{v_1} = a \vee b$$

$$\overline{v_4} = \neg a \vee \neg b$$

$$\overline{v} = (a \vee b) \wedge (\neg a \vee \neg b)$$

OPPURE

$$\neg a \leftrightarrow b \equiv (\neg a \rightarrow b) \wedge (b \rightarrow \neg a)$$

$$\equiv (a \vee b) \wedge (\neg b \vee \neg a)$$

ENC

$$\equiv ((a \vee b) \wedge \neg b) \vee ((a \vee b) \wedge \neg a) \equiv$$

$$\equiv (a \wedge \neg b) \vee \cancel{(b \wedge \neg b)} \vee \cancel{(a \wedge \neg a)} \vee (b \wedge \neg a)$$

b) $a \wedge (b \vee c)$

c) $((a \vee b) \wedge c) \vee d$

d) $a \vee b$

b)

- $(\neg a \vee b \vee c) \wedge (a \vee \neg b \vee \neg c) \wedge (a \vee \neg b \vee c) \wedge (a \vee b \vee \neg c) \wedge (a \vee b \vee c)$
- $(a \wedge b) \vee (a \wedge c)$

c) • $(a \wedge c) \vee (b \wedge c) \vee d$

- $(\neg a \vee b \vee \neg d) \wedge (c \vee \neg d)$

d) $(a \vee b) \wedge (a \vee \neg a)$

$(a \vee b)$

e) $(a \vee b) \rightarrow c$

f) $a \wedge (a \rightarrow b)$

g) • $\neg(a \vee b) \quad \text{E} \quad \neg a \wedge \neg b$

• $\neg(a \wedge b) \quad \text{E} \quad \neg a \vee \neg b$