

# PHD COURSE ON CATEGORY THEORY

## HOMEWORK SET 1

Given name: \_\_\_\_\_

Family name: \_\_\_\_\_

- To be handed in: Tuesday, 24th of January, at the beginning of the lecture.
- Solutions can be written either in Italian or English.
- Working together is encouraged, but must be acknowledged.
- **Good work!**

**Exercise 1.** Let  $\mathcal{C}$  be a category. We used the template  $\begin{array}{c} \bullet \\ \downarrow \\ S \end{array}$  to form the category  $(\mathcal{C} \downarrow S)$ . Show that

we if use the template  $\begin{array}{cc} \bullet & \bullet \\ & \searrow \swarrow \\ & S \end{array}$  we obtain a category in a similar way.

**Exercise 2.** Show that any arrow whose domain is the terminal object is monic.

**Exercise 3.** Prove that a morphism that is both a monomorphism and a split epimorphism is necessarily an isomorphism.

**Exercise 4.** Show that if a category has a terminal object  $1$  and products, then for every object  $A$ ,  $A \times 1 \cong A$

**Exercise 5.** Suppose the category  $\mathcal{C}$  has all binary products and all equalizers. Show that  $\mathcal{C}$  has all pullbacks.

**Exercise 6.** What is a functor between groups, regarded as one-object categories?

**Exercise 7.** Let  $\mathcal{R}$  be a monoid viewed as a category. Can you describe a covariant functor from  $\mathcal{R}$  into  $\mathcal{Set}$  as a classical mathematical structure?