Many-Valued Logics (Autumn 2013)

First homework assignment

- Deadline: 12 September — at the **beginning** of class.
- Grading is from 0 to 100 points; you get 10 points for free.
- Success!

30 pt

Exercise 1 (Residua)
Prove that for a t-norm $*\colon [0,1]^2 \to [0,1]$ to be residuated it is necessary and sufficient that $*$ is left-continuous. In other words, for any t-norm $*$ there exists an operation $\Rightarrow$ such that

$$x * y \leq z \qquad \text{IFF} \qquad x \leq y \Rightarrow z.$$

if, and only if, $*$ satisfies

$$\text{if} \quad \sup\{x_i \mid i \in I\} = x \quad \text{then} \quad \sup\{x_i * y \mid i \in I\} = x * y.$$

30 pt

Exercise 2 (Axiomatization)
Consider the following axiom

$$\text{(A3')} \qquad (\varphi \to (\psi \to \chi)) \to (\psi \to (\varphi \to \chi))$$

Prove that axiom (A3) is derivable from the remaining axioms in Definition 2.2.4 of *Metamathematics of Fuzzy Logic* plus (A3').

30 pt

Exercise 3 (Subdirectly irreducible BL-algebras)
Recall that a *subdirect product* of a family of algebras $\{A_i \mid i \in I\}$ is an algebra A such that:

1. there exists an embedding $e\colon A \to \prod_{i\in I} A_i$
2. for any $i \in I$, the composition $\pi_i \circ e\colon A \to A_i$ is surjective. Where each π_i is the canonical projection from $\prod_{i\in I} A_i$ into A_i.

An algebra A is said *sudirectly irreducible* if whenever it is isomorphic to a subdirect product of a family of algebras $\{A_i \mid i \in I\}$, the algebra A is isomorphic to one of the A_i.

Prove that every subdirectly irreducible BL-algebra is linearly ordered.