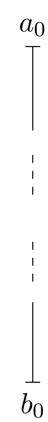
Many-Valued Logics (Autumn 2013)

Second homework assignment

- Deadline: 19 September — at the **beginning** of class.
- Grading is from 0 to 100 points; you get 10 points for free.
- Success!

30 pt **Exercise 1.** (Embedding into the real interval)
Consider the BL-algebra $\langle C, *, \Rightarrow, \wedge, \vee, 0, 1\rangle$ where

1. $C = \{a_n \mid n \in \mathbb{N}\} \bigcup \{b_n \mid n \in \mathbb{N}\}$ where all a_i and b_j are different, and $a_0 = 1$ and $b_0 = 0$.
2. The order is given by
   (a) $\forall m, n \in \mathbb{N}\ b_m < a_n$,
   (b) $\forall m, n \in \mathbb{N}\ a_m < a_n$ if, and only if, $m > n$,
   (c) $\forall m, n \in \mathbb{N}\ b_m < b_n$ if, and only if, $m < n$.
3. $*$ is defined as
   (a) $\forall m, n \in \mathbb{N}\ b_m * b_n = 0$,
   (b) $\forall m, n \in \mathbb{N}\ a_m * a_n = a_{m+n}$,
   (c) $\forall m, n \in \mathbb{N}\ a_m * b_n = \begin{cases} b_{n-m} & \text{if } m \leq n \\ 0 & \text{otherwise.} \end{cases}$
4. $\Rightarrow$ is the residuum of $*$. The operations $\wedge$ and $\vee$ are, respectively, minimum and maximum w.r.t. the order defined above.



Prove that C cannot be homomorphically embedded in any standard BL-algebra.

20 pt **Exercise 2.** (Finitely generated BL-algebras)
Recall that an algebra A is called *finitely generated* if there exists a finite set $\{a_1, ..., a_n\} \subseteq A$ such that for any $a \in A$ there exists a term $t(x_1, ..., x_n)$ in the language of A such that $a = t(a_1, ..., a_n)$.
Prove that any linearly ordered finitely generated BL-algebra is the ordinal sum of finitely many **irreducible** components.

30 pt **Exercise 3.** (Łukasiewicz t-norm)
Prove that the Łukasiewicz t-norm is the only continuous t-norm with a continuous residuum.