Many-Valued Logics (Autumn 2013)

Third homework assignment

- Deadline: 26 September — at the **beginning** of class.

- Grading is from 0 to 100 points; you get 10 points for free.

- Success!

30 pt

Exercise 1. (Matrix Semantics)
Prove that the class of all formula matrices form a matrix semantics for any (finitary and structural) deductive system $\mathcal{S}$.

30 pt

Exercise 2. (Algebraic Semantics)
Prove that if a (finitary and structural) deductive system has an algebraic semantics, then there exists a largest algebraic semantics.

30 pt

Exercise 3. (The lattice of theories)
Let $H\colon \mathcal{T}h(K) \to \mathcal{T}h(\mathcal{S})$ be defined, for any equational theory Θ, by

$$H(\Theta) := \{\varphi \in Fm \mid \delta(\varphi) = \varepsilon(\varphi) \in \Theta\}$$

Let $\Omega\colon \mathcal{T}h(\mathcal{S}) \to \mathcal{T}h(K)$ be defined, for any theory T, by

$$\Omega(T) := Cn_K(\{\delta(\varphi) = \varepsilon(\varphi) \mid \varphi \in T\})$$

Prove that H, Ω form a covariant Galois connection i.e.,

$$\Omega(T) \subseteq \Theta \text{ if, and only if, } T \subseteq H(\Theta).$$