Many-Valued Logics (Autumn 2013)

Fifth homework assignment

- Deadline: 10 October — at the **beginning** of class.
- Grading is from 0 to 100 points; you get 10 points for free.
- Success!

30 pt **Exercise 1.** (Dual algebra)
Prove that if W is a residuated frame, then W^+ is a complete residuated lattice.

30 pt **Exercise 2.** (Analytic rules)
Prove that the following equation is equivalent to an analytic quasi-equation.

$$x \cdot y \cdot x \leq y \cdot x$$

You can get inspired by the *restructuring* and *cutting* procedure on section 4 of CM5, but you should give a direct proof here.

30 pt **Exercise 3.** (Dense extensions)
Let P be a partially ordered set and C be a doubly-dense extension of P i.e., there is an embedding $i : P \hookrightarrow C$ and for any $c \in C$ there exist $X, Y \subseteq P$ such that $c = \bigwedge i[X] = \bigvee i[Y]$. Prove that i preserves all (possibly infinite) existing meets and joins of P.